Complexity of Computerized Turnstiles

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Abstract

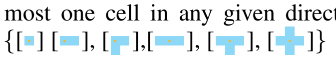
Are turnstiles computationally hard? Pretty much.

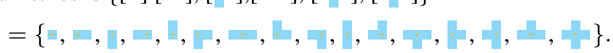
1. Introduction

Turnstiles have been an object of interest to computer scientists and walkers alike for days[Diary]. They are the object of interest in the game *Kwirk*, known in Japan as *Puzzle Boy*, in which a player navigates an agent through a puzzle to reach a goal. Each puzzle can contain unmovable bricks, movable boxes, holes which can be filled with boxes of the right size, and movable turnstiles; the agent can push movable objects, but movable objects cannot move other movable objects. One[1] can easily prove[DB] that a generalized puzzle is PSPACE-complete by a reduction from nondeterministic constraint logic[GPC]. The computational complexities of various variations of this game were asked by Aaron Williams at the 32nd Bellairs Winter Workshop on Computational Geometry in Holetown, Barbados earlier this year. However, we instead consider *computerized turnstile puzzles* which have clear applications to those who encounter turnstiles in their everyday lives. We formally define one in Section 2.1, and discuss its hardness and application in Section 3. We end with a list of open problems in Section 4.

2. Turnstile Puzzles

We follow Kwirk's model of computation. A turnstile is formally defined to be a subset of cells in a + shape that includes the center cell and at most one cell in any given direction. The set of all turnstiles is





The agent can push turnstiles 90° to push turnstiles clockwise or counterclockwise. If pushing one arm results in another arm pushing the agent, then the agent moves two spaces instead of none. Of course, turnstiles can only move through empty spaces. A *turnstile puzzle* is one which only uses unmovable objects and turnstiles. Figure 1 below is an example level.

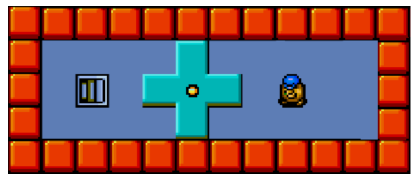


Figure 1. Example level

It can be solved, for example, by moving up (down) once, left five times, and down (up) once.

2.1 Computerized Turnstile Puzzles (C′TP)

Computerized turnstile puzzles, henceforth known as C′TPs, are similar to turnstiles in that they can contain any number of unmovable bricks, but they differ in that C′TPs contain at most one *computerized turnstile*. The computerized turnstile contains a computer with the same problem on each arm, and can only be pushed once the agent solves the problem. The decision problem can be formulated as follows.

Instance 1. *decidable C-hard problem C_0 for some complexity class C; computerized turnstile puzzle layout, which is solvable when viewed as a turnstile puzzle*

Question 1. *Is there a sequence of moves that drives the agent to the goal?*

Suppose the arms on the turnstile in Figure 1 were equipped with the boolean satisfiability problem $(x_1 \vee \overline{x_2}) \wedge (x_1 \vee x_2)$. This C′TP can be solved by moving up (down) once, left twice, answering $x_1 = \texttt{True}, x_2 = \texttt{False}$, moving left thrice, and down (up) once. The difficulty of the general problem is proven in the next section.

3. Result

3.1 Hardness

Theorem 1. *C′TP is C-hard.*

Proof. I reduce the C-hard problem C_0 to C′TP. First, solve C_0 on any of the turnstile's arms. Then, push turnstile and walk to goal. C′TP is thus as hard as C. □

Corollary 1. *C′TP with SAT is NP-complete, and C′TP with QBF is PSPACE-complete.*

[1] In the words of Pierre de Fermat,"I have discovered a truly marvelous demonstration of this proposition that this margin is too narrow to contain."

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Proof. Both hardness proofs follow are direct applications of the theorem above. C′TP with SAT is in NP since there exists an algorithm in NP that solves SAT; similarly, C′TP with QBF is in PSPACE. □

3.2 Application

C′TP can be applied anytime an agent needs to solve a problem to unlock a turnstile to get to a goal. A few examples are listed below:

- New York City Subway with generalized problem needed to be solved to unlock turnstile
- San Diego Zoo with generalized problem needed to be solved to push turnstile
- Museum of Science and Industry with generalized problem needed to be solved to push turnstile
- Chicago L with generalized problem needed to be solved to push turnstile
- a revolving door in Phoenix with generalized problem needed to be solved to push turnstile

4. Conclusion

The authors would like to remind the reader that most turnstiles in life can be unlocked by paying a small fee. It could be a lot harder: Can you imagine having to solve an instance of 3SAT with 57 clauses just to get to work one day?

References

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